2.4 Simplifying Fractions

2.4 OBJECTIVES

1. Determine whether two fractions are equivalent
2. Use the fundamental principle to simplify fractions

It is possible to represent the same portion of the whole by different fractions. Look at Figure 1, representing $\frac{3}{6}$ and $\frac{1}{2}$. The two fractions are simply different names for the same number. They are called **equivalent fractions** for this reason.

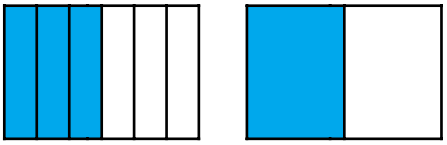

Figure 1

Any fraction has a large number of equivalent fractions. For instance, $\frac{2}{3}$, $\frac{4}{6}$, and $\frac{6}{9}$ are all equivalent fractions because they name the same part of a unit. This is illustrated in Figure 2.

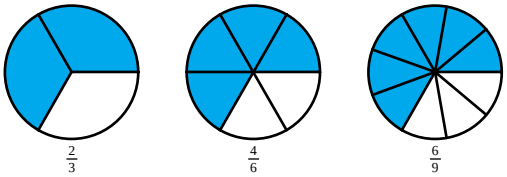

Figure 2

Many more fractions are equivalent to $\frac{2}{3}$. All these fractions can be used interchangeably. An easy way to find out if two fractions are equivalent is to use cross products.

$\frac{a}{b} = \frac{c}{d}$ We call $a \times d$ and $b \times c$ the **cross products.**

Rules and Properties: Testing for Equality

If the cross products for two fractions are equal, the two fractions are equivalent.

Example 1

Identifying Equivalent Fractions Using Cross Products

(a) Are $\frac{3}{24}$ and $\frac{4}{32}$ equivalent fractions? The cross products are 3×32, or 96, and 24×4, or 96. Because the cross products are equal, the fractions are equivalent.

(b) Are $\frac{2}{5}$ and $\frac{3}{7}$ equivalent fractions?

The cross products are 2×7 and 5×3.

$2 \times 7 = 14$ and $5 \times 3 = 15$

Because $14 \neq 15$, the fractions are *not* equivalent.

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CHECK YOURSELF 1

(a) Are $\frac{3}{8}$ and $\frac{9}{24}$ equivalent fractions? **(b)** Are $\frac{7}{8}$ and $\frac{8}{9}$ equivalent fractions?

In writing equivalent fractions, we use the following important principle.

Rules and Properties: The Fundamental Principle of Fractions

For the fraction $\frac{a}{b}$ and any nonzero number c,

$$\frac{a}{b} = \frac{a \div c}{b \div c}$$

NOTE We are really dividing by $\frac{c}{c}$, or 1, and dividing by 1 does not change the value of a number.

The Fundamental Principle of Fractions tells us that we can divide the numerator and denominator by the same nonzero number. The result is an equivalent fraction. For instance,

NOTE Divide the numerator and denominator by 2, 3, and 4.

$$\frac{2}{4} = \frac{2 \div 2}{4 \div 2} = \frac{1}{2} \qquad \frac{3}{6} = \frac{3 \div 3}{6 \div 3} = \frac{1}{2} \qquad \frac{4}{8} = \frac{4 \div 4}{8 \div 4} = \frac{1}{2}$$

NOTE Divide the numerator and denominator by 5, 6, and 7.

$$\frac{5}{10} = \frac{5 \div 5}{10 \div 5} = \frac{1}{2} \qquad \frac{6}{12} = \frac{6 \div 6}{12 \div 6} = \frac{1}{2} \qquad \frac{7}{14} = \frac{7 \div 7}{14 \div 7} = \frac{1}{2}$$

Let's see how this is applied.

Simplifying a fraction or *reducing a fraction to lower terms* means finding an equivalent fraction with a *smaller* numerator and denominator than those of the original fraction. Dividing the numerator and denominator by the same nonzero number will do exactly that.

Consider Example 2.

Example 2

Simplifying Fractions

Simplify each fraction.

NOTE We apply the fundamental principle to divide the numerator and denominator by 5.

(a) $\frac{5}{15} = \frac{5 \div 5}{15 \div 5} = \frac{1}{3}$

$\frac{5}{15}$ and $\frac{1}{3}$ are equivalent fractions. Check this by finding the cross products.

NOTE We divide the numerator and denominator by 2.

(b) $\frac{4}{8} = \frac{4 \div 2}{8 \div 2} = \frac{2}{4}$

$\frac{4}{8}$ and $\frac{2}{4}$ are equivalent fractions.

CHECK YOURSELF 2

Write two fractions that are equivalent to $\frac{30}{45}$.

(a) Divide the numerator and denominator by 5.

(b) Divide the numerator and denominator by 15.

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We say that a fraction is in **simplest form,** or in **lowest terms,** if the numerator and denominator have no common factors other than 1. This means that the fraction has the smallest possible numerator and denominator.

In Example 2, $\frac{1}{3}$ is in simplest form because the numerator and denominator have no common factors other than 1. The fraction is in lowest terms.

$\frac{2}{4}$ is *not* in simplest form. Do you see that $\frac{2}{4}$ can also be written as $\frac{1}{2}$?

NOTE In this case, the numerator and denominator are *not* as small as possible. The numerator and denominator have a common factor of 2.

To write a fraction in simplest form or to *reduce a fraction to lowest terms,* divide the numerator and denominator by their greatest common factor (GCF).

Example 3

Simplifying Fractions

Write $\frac{10}{15}$ in simplest form.

From our work earlier in this chapter, we know that the greatest common factor of 10 and 15 is 5. To write $\frac{10}{15}$ in simplest form, divide the numerator and denominator by 5.

$$\frac{10}{15} = \frac{10 \div 5}{15 \div 5} = \frac{2}{3}$$

The resulting fraction, $\frac{2}{3}$, is in lowest terms.

CHECK YOURSELF 3

Write $\frac{12}{18}$ in simplest form by dividing the numerator and denominator by the GCF.

Many students prefer another method of reducing fractions, which uses the prime factorizations of the numerator and denominator. Example 4 uses this method.

Example 4

Factoring to Simplify a Fraction

(a) Simplify $\frac{24}{42}$.

To simplify $\frac{24}{42}$, factor.

$$\frac{24}{42} = \frac{\overset{1}{\cancel{2}} \times 2 \times 2 \times \overset{1}{\cancel{3}}}{\underset{1}{\cancel{2}} \times \underset{1}{\cancel{3}} \times 7} = \frac{4}{7}$$

NOTE From the prime factorization of 24 and 42, we divide by the common factors of 2 and 3.

Note: The numerator of the simplified fraction is the *product* of the prime factors remaining in the numerator after the division by 2 and 3.

(b) Simplify $\frac{120}{180}$.

To reduce $\frac{120}{180}$ to lowest terms, write the prime factorizations of the numerator and denominator. Then divide by any common factors.

$$\frac{120}{180} = \frac{\overset{1}{\cancel{2}} \times \overset{1}{\cancel{2}} \times 2 \times \overset{1}{\cancel{3}} \times \overset{1}{\cancel{5}}}{\underset{1}{\cancel{2}} \times \underset{1}{\cancel{2}} \times \underset{1}{\cancel{3}} \times 3 \times \underset{1}{\cancel{5}}} = \frac{2}{3}$$

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CHECK YOURSELF 4

Write each of the following fractions in simplest form.

(a) $\frac{60}{75}$ **(b)** $\frac{210}{252}$

There is another way to organize your work in simplifying fractions. It again uses the fundamental principle to divide the numerator and denominator by any common factors. Let's illustrate with the fractions considered in Example 4.

Example 5

Using Common Factors to Simplify Fractions

(a) $\frac{24}{42} = \frac{\overset{12}{\cancel{24}}}{\underset{21}{\cancel{42}}} = \frac{\overset{4}{\cancel{12}}}{\underset{7}{\cancel{21}}} = \frac{4}{7}$

Divide by the common factor of 2.

Divide by the common factor of 3.

The original numerator and denominator are divisible by 2, and so we divide by that factor to arrive at $\frac{12}{21}$. Our divisibility tests tell us that a common factor of 3 still exists. (Do you remember why?) Divide again for the result $\frac{4}{7}$, which is in lowest terms.

Note: If we had seen the GCF of 6 at first, we could have divided by 6 and arrived at the same result in one step.

(b) $\frac{120}{180} = \frac{\overset{2}{\cancel{120}}}{\underset{3}{\cancel{180}}} = \frac{2}{3}$

Our first step is to divide by the common factor of 6. We then have $\frac{20}{30}$. There is still a common factor of 10, so we again divide.

Again, we could have divided by the GCF of 60 in one step if we had recognized it.

CHECK YOURSELF 5

Using the method of Example 5*, write each of the fractions in simplest form.*

(a) $\frac{60}{75}$ **(b)** $\frac{84}{196}$

CHECK YOURSELF ANSWERS

1. **(a)** Yes; **(b)** No **2.** **(a)** $\frac{6}{9}$; **(b)** $\frac{2}{3}$

3. 6 is the GCF of 12 and 18, so $\frac{12}{18} = \frac{12 \div 6}{18 \div 6} = \frac{2}{3}$

4. **(a)** $\frac{60}{75} = \frac{2 \times 2 \times \overset{1}{\cancel{3}} \times \overset{1}{\cancel{5}}}{\underset{1}{\cancel{3}} \times \underset{1}{\cancel{5}} \times 5} = \frac{4}{5}$; **(b)** $\frac{210}{252} = \frac{\overset{1}{\cancel{2}} \times \overset{1}{\cancel{3}} \times 5 \times \overset{1}{\cancel{7}}}{\underset{1}{\cancel{2}} \times 2 \times 3 \times \underset{1}{\cancel{3}} \times \underset{1}{\cancel{7}}} = \frac{5}{6}$

5. **(a)** Divide by the common factors of 3 and 5, $\frac{60}{75} = \frac{4}{5}$

(b) Divide by the common factors of 4 and 7, $\frac{84}{196} = \frac{3}{7}$

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2.4 Exercises

Name _____

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Are the pairs of fractions equivalent?

1. $\frac{1}{3}, \frac{3}{5}$

2. $\frac{3}{5}, \frac{9}{15}$

3. $\frac{1}{7}, \frac{4}{28}$

4. $\frac{2}{3}, \frac{3}{5}$

5. $\frac{5}{6}, \frac{15}{18}$

6. $\frac{3}{4}, \frac{16}{20}$

7. $\frac{2}{21}, \frac{4}{25}$

8. $\frac{20}{24}, \frac{5}{6}$

9. $\frac{2}{7}, \frac{3}{11}$

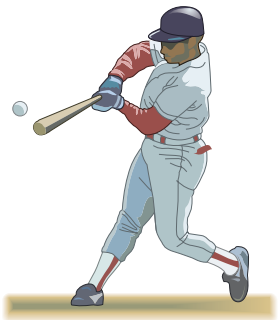
10. $\frac{12}{15}, \frac{36}{45}$

11. $\frac{16}{24}, \frac{40}{60}$

12. $\frac{15}{20}, \frac{20}{25}$

13. Test score. On a test of 72 questions, Sam answered 54 correctly. On another test Sam answered 66 correct out of 88. Did Sam get the same portion of each test correct?

14. Batting average. Jeff Bagwell of the Houston Astros has 104 hits in 325 times at bat. Matt Williams of the Arizona Diamondbacks has 88 hits in 275 times at bat. Do they have the same batting average?



Write each fraction in simplest form.

15. $\frac{8}{12}$

16. $\frac{12}{15}$

17. $\frac{10}{14}$

18. $\frac{15}{50}$

ANSWERS

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____
7. _____
8. _____
9. _____
10. _____
11. _____
12. _____
13. _____
14. _____
15. _____
16. _____
17. _____
18. _____

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ANSWERS

19. _____

20. _____

21. _____

22. _____

23. _____

24. _____

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26. _____

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28. _____

29. _____

30. _____

31. _____

32. _____

33. _____

34. _____

35. _____

36. _____

37. _____ 38. _____

39. _____ 40. _____

41. _____ 42. _____

43. _____ 44. _____

19. $\frac{12}{18}$

20. $\frac{28}{35}$

21. $\frac{35}{40}$

22. $\frac{21}{24}$

23. $\frac{11}{44}$

24. $\frac{10}{25}$

25. $\frac{12}{36}$

26. $\frac{18}{48}$

27. $\frac{24}{27}$

28. $\frac{30}{50}$

29. $\frac{32}{40}$

30. $\frac{17}{51}$

31. $\frac{75}{105}$

32. $\frac{62}{93}$

33. $\frac{48}{60}$

34. $\frac{48}{66}$

35. $\frac{105}{135}$

36. $\frac{54}{126}$

37. $\frac{66}{110}$

38. $\frac{280}{320}$

39. $\frac{16}{21}$

40. $\frac{21}{32}$

41. $\frac{31}{52}$

42. $\frac{42}{55}$

Solve the following applications.

43. Coins. A quarter is what fractional part of a dollar? Simplify your result.

44. Coins. A dime is what fractional part of a dollar? Simplify your result.

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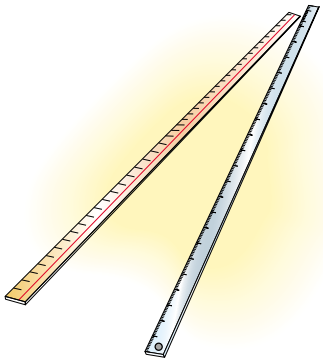
45. Time. What fractional part of an hour is 15 minutes (min)? Simplify your result.

46. Time. What fractional part of a day is 6 hours (h)? Simplify your result.

47. Length. A meter is equal to 100 centimeters (cm). What fractional part of a meter is 70 cm? Simplify your result.



48. Length. A kilometer is equal to 1000 meters (m). What fractional part of a kilometer is 300 m? Simplify your result.

49. Auto repairs. Susan did a tune-up on her automobile. She found that two of her eight spark plugs were fouled. What fraction represents the number of fouled plugs? Reduce to lowest terms.

50. Testing. Samantha answered 18 of 20 problems correctly on a test. What fractional part did she answer correctly? Reduce your answer to lowest terms.

51. Baseball. The local baseball team won 36 of the 58 games they played. What fractional part did they win? Reduce your answer to lowest terms.

52. Salary. Sharon earned $250 at her after-school job. A new bike costs $120. What fractional part of her money will remain after she purchases the bike? Reduce your answer to lowest terms.

53. A student is attempting to reduce the fraction $\frac{8}{12}$ to lowest terms. He produces the following argument:

$$\frac{8}{12} = \frac{4+4}{8+4} = \frac{4}{8} = \frac{1}{2}$$

What is the fallacy in this argument? What is the correct answer?

ANSWERS

45. _____

46. _____

47. _____

48. _____

49. _____

50. _____

51. _____

52. _____

53. _____

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54. _____

55. _____

56. _____

54. Can any of the following fractions be simplified?

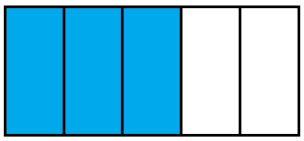
(a) $\frac{824}{73}$ **(b)** $\frac{59}{11}$ **(c)** $\frac{135}{17}$

What characteristic do you notice about the denominator of each fraction? What rule would you make up based on your observations?

55. Consider the figure at the right.

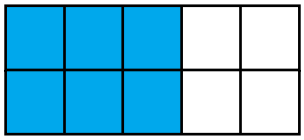
(a) Give the fraction that represents the shaded region.

(a)

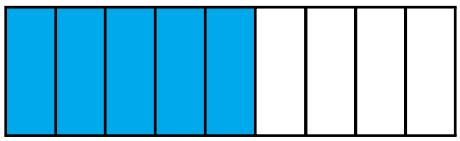


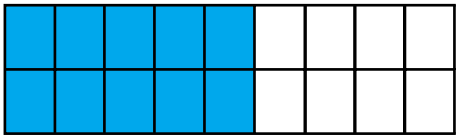
(b) Draw a horizontal line through the figure, as shown. Now give the fraction representing the shaded region.

(b)



56. Repeat exercise 55 using these figures.





Answers

1. $1 \times 5 = 5$; $3 \times 3 = 9$. The fractions are not equivalent. **3.** Yes
5. Yes **7.** No **9.** No
11. $16 \times 60 = 960$, and $24 \times 40 = 960$. The fractions are equivalent. **13.** Yes
15. $\frac{2}{3}$ **17.** $\frac{5}{7}$ **19.** $\frac{2}{3}$ **21.** $\frac{7}{8}$ **23.** $\frac{1}{4}$ **25.** $\frac{1}{3}$ **27.** $\frac{8}{9}$
29. $\frac{4}{5}$ **31.** $\frac{5}{7}$ **33.** $\frac{4}{5}$ **35.** $\frac{7}{9}$ **37.** $\frac{3}{5}$ **39.** $\frac{16}{21}$ **41.** $\frac{31}{52}$
43. $\frac{1}{4}$ **45.** $\frac{1}{4}$ **47.** $\frac{7}{10}$ **49.** $\frac{1}{4}$ **51.** $\frac{18}{29}$ **53.**
55. **(a)** $\frac{3}{5}$; **(b)** $\frac{6}{10}$

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Using Your Calculator to Simplify Fractions

If you have a calculator that supports fraction arithmetic, you should learn to use it to check your work. We'll look at two different types of these calculators.

Scientific Calculator

Scientific calculators include the TI-34, the Casio fx-250 or fx-115, and the Sharp 506g or 509g.

Before doing Example 1, find the button on your scientific calculator that is labeled **a b/c**. This is the button that will be used to enter fractions.

Example 1

Using a Scientific Calculator to Simplify Fractions

Simplify the fraction $\frac{24}{68}$.

There are four steps in simplifying fractions using a scientific calculator.

1. Enter the numerator, 24.
2. Press the **a b/c** key.
3. Enter the denominator, 68.
4. Press =.

The calculator will display the simplified fraction, $\frac{6}{17}$.

CHECK YOURSELF 1

Simplify the fraction $\frac{51}{81}$.

Graphing Calculator

Let's simplify the same fraction, $\frac{24}{68}$, using a graphing calculator, such as the TI-82 or TI-83:

1. Enter the fraction as a division problem: 24 ÷ 68. The calculator will display 24/68.
2. Press the **MATH** key.
3. Select **1: ▶ Frac**.
4. Press Enter.

The calculator displays the simplified fraction, $\frac{6}{17}$.

The graphing calculator is particularly useful for simplifying fractions with large values in the numerator and denominator.

NOTE Some scientific calculators cannot handle denominators larger than 999.

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Example 2

Using a Graphing Calculator to Simplify Fractions

Simplify $\frac{546}{637}$.

Using our calculator, we find that

$$\frac{546}{637} = \frac{6}{7}$$

CHECK YOURSELF 2

Simplify $\frac{649}{885}$.

CHECK YOURSELF ANSWERS

1. $\frac{17}{27}$ **2.** $\frac{11}{15}$

Name _____

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ANSWERS

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____
7. _____
8. _____
9. _____

Calculator Exercises

Use your calculator to simplify the following fractions.

1. $\frac{28}{40}$

2. $\frac{121}{132}$

3. $\frac{96}{144}$

4. $\frac{385}{605}$

5. $\frac{445}{623}$

6. $\frac{153}{255}$

7. $\frac{299}{391}$

8. $\frac{152}{209}$

9. $\frac{289}{459}$

Answers

1. $\frac{7}{10}$ **3.** $\frac{2}{3}$ **5.** $\frac{5}{7}$ **7.** $\frac{13}{17}$ **9.** $\frac{17}{27}$

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